7.4. , 14.5. ← - písemky (v F1)
↖ elektrostatika ← el. obvody a magnetismus
střídavý proud

skripta 1991 Bakule a spol. : Příklady z el. a mag.

VEKTOROVÁ ANALÝZA

$|\vec{a}| = a$

$\vec{a} \cdot \vec{b} = a_1 b_1 + \cdots = ab \cos\alpha = \vec{b} \cdot \vec{a}$

$\vec{a}(\vec{b} \cdot \vec{c}) = (\vec{a} \cdot \vec{b})\vec{c}$

$c = \vec{a} \times \vec{b} = \vec{n} \cdot ab \sin\alpha$

$c_x = a_2 b_3 - a_3 b_2$

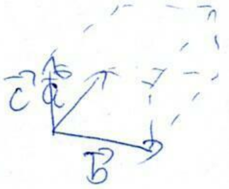
$c_y = a_3 b_1 - a_1 b_3$

$c_z = a_1 b_2 - a_2 b_1$

$$\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{matrix}$$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

smíšený součin $\vec{c} \cdot (\vec{a} \times \vec{b})$ objem



$dl_i = \overset{h_i(q_1, q_2, q_3)}{\cancel{h_1 h_2 h_3}} \, dq_i$

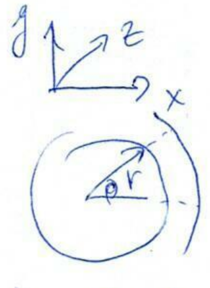
↑ Laméovy koef.

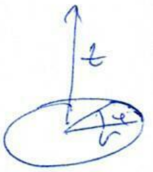
	j	k	l
dl_i	$1dx$	$1dy$	$1dz$
dl_i	$1dr$	$1dz$	$r d\varphi$
dl_i	$1dr$	$r d\vartheta$	$r \sin\vartheta d\varphi$

$dS = r \, dr \, d\varphi$

v sf. $dV = r^2 \sin\vartheta \, dr \, d\vartheta \, d\varphi$

v kar. $dV = dx \, dy \, dz$





$dl_r = dr$

$dl_\varphi = r \, d\varphi$

$\frac{x}{r} = \sin\vartheta$

~~tez~~ $x = r \sin\vartheta$

$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$

gradient $\text{grad} f = \vec{F} = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$

divergence $\text{div} \vec{F} = g = \nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$

rotace (jen ve 3D) $\text{rot} \vec{F} = \vec{G} = \underset{\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}}{\nabla} \times \underset{\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}}{\vec{F}} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\cancel{\partial z}}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$

Laplaceův op. $\Delta = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$\vec{F} = (2y, -2x, 0)$

$\boxed{\text{rot grad} f = 0} = \nabla \times \nabla f$

$\boxed{\text{div rot} \vec{G} = 0}$

$\text{rot} \vec{F} = 0 \Rightarrow \text{ex.} \vec{F} = \text{grad} f$

pole je potenciální

$\text{div} \vec{G} = 0 \Rightarrow \text{ex.} \vec{G} = \text{rot} \vec{G}_1$

solenoidní pole

Stokesova v. $\oint \vec{F}\cdot d\vec{l} = \int \text{rot } \vec{F}\cdot d\vec{S}$

Gaussova v. $\oint \vec{F}\cdot d\vec{S} = \int \text{div } \vec{F}\, dV$

Př. Spočítejte divergenci a rotaci vektorových polí. Je-li rot $\vec{F}=0$, najděte sk. pole takové, aby $\vec{F} = \text{grad} f$

- $\vec{F} = (2y, 2x+3z, 3y)$

$$\text{div } \vec{F} = \nabla\cdot\vec{F} = \left(\frac{\partial 2y}{\partial x} + \frac{\partial(2x+3z)}{\partial y} + \frac{\partial 3y}{\partial z}\right)$$

$$= 0+0+0 = 0$$

$$\text{rot } \vec{F} = \nabla\times\vec{F} = \left(\frac{\partial 3y}{\partial y} - \frac{\partial(2x+3z)}{\partial z}, \frac{\partial 2y}{\partial z} - \frac{\partial 3y}{\partial x}, \frac{\partial(2x+3z)}{\partial x} - \frac{\partial 2y}{\partial y}\right)$$

$$\begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix}\begin{pmatrix}2y\\ 2x+3z\\ 3y\end{pmatrix} = (0, 0, 0)$$

$$\vec{F} = \text{grad } f = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$3y = \frac{\partial f}{\partial z}$

$\int 3y\,\partial z = \int \partial f$

$3yz = f_z$

3

$2y = \frac{\partial f}{\partial x}$

$\int 2y\,\partial x = \int \partial f$

$2yx = f_x$

1.

$2x+3z = \frac{\partial f}{\partial y}$

$(2x+3z)\partial y = \partial f$

$\int (2x+3z)\partial y = \int \partial f$

$(2x+3z)y = f_y$

2.

~~$f = 3yz + 2yx +$~~ $f = 2xy + 2xy + 3yz + 3yz$

$= 4xy + 6yz$

$= 2xy + 3yz$

Spočítejte gradient pole.

$\vec{r} = (x, y, z) \qquad r = \sqrt{x^2+y^2+z^2}$

- $\text{grad } r = \left(\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y}, \frac{\partial r}{\partial z}\right) = \left(\frac{1\cdot 2x}{2\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{-\text{"}-}}\right) = \frac{\vec{r}}{r}$

- $\text{grad } r^2 = \left(\frac{\partial r^2}{\partial x}, \frac{\partial r^2}{\partial y}, \frac{\partial r^2}{\partial z}\right) = (2x, 2y, 2z) = 2\vec{r}$

- $\text{grad } \frac{1}{r} = \left(\frac{\partial \frac{1}{r}}{\partial x}, \dots\right) = -\frac{1}{r^3}\cdot\vec{r}$

$$\frac{\partial \frac{1}{\sqrt{x^2+y^2+z^2}}}{\partial x}, (x^2+y^2+z)^{-\frac{1}{2}} = -\frac{1}{2}(x^2+y^2+z)^{-\frac{3}{2}}\cdot 2x = -(x^2+y^2+z)^{-\frac{3}{2}}\cdot x$$

- $\text{grad } \frac{1}{r^3} = -\frac{3\vec{r}}{r^5}$

$$\frac{\partial \frac{1}{r^3}}{\partial x} = \frac{\partial (x^2+y^2+z^2)^{-\frac{3}{2}}}{\partial x} = -\frac{3}{2}(x^2+y^2+z^2)^{-\frac{5}{2}}\cdot 2x = -3x\cdot r^{-\frac{5}{2}}$$

grün - gauss $\oint f\,d\vec{S} = \int \nabla f\,dV$

$\vec{r} = (x, y, z)$ $r = (x^2+y^2+z^2)^{\frac{1}{2}}$ $\vec{c} = (c_1, c_2, c_3)$

$$\nabla(\vec{c}\cdot\vec{r}) = \nabla(c_1 x + c_2 y + c_3 z) = (c_1, c_2, c_3) = \vec{c}$$

$$\nabla\left(\frac{\vec{c}\cdot\vec{r}}{r}\right)_x = \frac{1}{r}\cdot\nabla(\vec{c}\vec{r}) + \nabla\left(\frac{1}{r}\right)\cdot(\vec{c}\cdot\vec{r}) = \frac{c}{r} - \frac{\vec{r}}{r^3}(\cdot\vec{c}\vec{r}) = \frac{r^2 c - (\vec{c}\cdot\vec{r})\cdot\vec{r}}{r^3}$$

Př.4.

$$\operatorname{div}\vec{r} = \left(\frac{\partial x}{\partial x}, \frac{\partial y}{\partial y}, \frac{\partial z}{\partial z}\right) = (1+1+1) = 3$$

$$\operatorname{rot}\vec{r} = \begin{pmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{pmatrix} \times \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \Big(\underset{0}{\underset{\parallel}{\frac{\partial z}{\partial y}}} - \underset{0}{\underset{\parallel}{\frac{\partial y}{\partial z}}}, \dots, \dots\Big) = \vec{0}$$

$$\operatorname{div}\frac{\vec{r}}{r} = \nabla\cdot\frac{\vec{r}}{r} = \left(\frac{\partial\frac{x}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial x} + \frac{\partial\frac{y}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial y} + \frac{\partial\frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial z}\right) = r^{-3}(r^2-x^2+r^2-y^2+r^2-z^2)$$

$$= 3r^{-1} - r^{-3}(x^2+y^2+z^2)$$
$$= 3r^{-1} - r^{-3}\cdot r^2 = 3r^{-1} - r^{-1}$$
$$= 2r^{-1}$$

$$\frac{\partial\frac{x}{(x^2+y^2+z^2)^{\frac{1}{2}}}}{\partial x} = \frac{1\cdot(x^2+y^2+z^2)^{\frac{1}{2}} - x\cdot\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-\frac{1}{2}}\cdot\cancel{2}x}{x^2+y^2+z^2} = (x^2+y^2+z^2)^{-\frac{1}{2}} - x^2(x^2+y^2+z^2)^{-\frac{3}{2}} = r^{-1} - x^2\cdot r^{-3}$$

$$\operatorname{rot}\frac{\vec{r}}{r} = \nabla\times\frac{\vec{r}}{r} = \frac{1}{r}(\nabla\times\vec{r}) + \left(\nabla\frac{1}{r}\right)\times\vec{r} = \frac{1}{r}\begin{pmatrix}0\\0\\0\end{pmatrix} - \frac{\vec{r}}{r^3}\times\vec{r} = \vec{0}$$

$$\operatorname{div}\frac{\vec{r}}{r^3} = \nabla\cdot\frac{\vec{r}}{r^3} = \frac{1}{r^3}(\nabla\cdot\vec{r}) + \left(\nabla\frac{1}{r^3}\right)\cdot\vec{r} = \frac{1}{r^3}\cdot 3 - \frac{3\vec{r}}{r^5}\cdot\vec{r} = \frac{3}{r^3} - \frac{3r^2}{r^5} = \frac{3}{r^3} - \frac{3}{r^3} = 0$$

$$\operatorname{rot}\frac{\vec{r}}{r^3} = \nabla\times\frac{\vec{r}}{r^3} = \left(\nabla\frac{1}{r^3}\right)\times\vec{r} + (\nabla\times\vec{r})\cdot\frac{1}{r^3} = -\frac{3\vec{r}}{r^5}\times\vec{r} + 0\cdot\frac{1}{r^3} = \vec{0}$$

ELEKTROSTATIKA

Př.1. 2 stejné částice náboji e $m = ?$, aby $\vec{F}_G = -\vec{F}_{el.}$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{\cancel{r^2}} = G\frac{m^2}{\cancel{r^2}}$$

$$m = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} = 1{,}86\cdot10^{-9}\,kg$$

$$\frac{m}{m_{el.}} = 2\cdot10^{21}$$

$\vec{F} = q\vec{E}$ ← intenzita

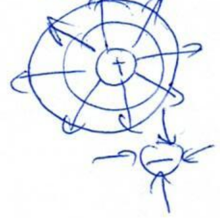
$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i\vec{r}_i}{r^3}$

1 náboj : $E = \frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$

$\varphi = -\nabla f$

$E = -\nabla\varphi$ ← potenciál

$\varphi = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$



Př. 11.9. $\frac{Q_2}{Q_1} = n$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{\sqrt{x^2+y^2+z^2}} - \frac{nQ_1}{\sqrt{(x-d)^2+y^2+z^2}}\right) = 0$$

$$\frac{\cancel{Q_1}}{\sqrt{x^2+y^2+z^2}} = \frac{n\cancel{Q_1}}{\sqrt{(x-d)^2+y^2+z^2}}$$

$$\sqrt{(x-d)^2+y^2+z^2} = n\sqrt{x^2+y^2+z^2}$$

$$(x-d)^2+y^2+z^2 = n^2(x^2+y^2+z^2)$$

$$x^2-2xd+d^2+y^2+z^2 = n^2x^2+n^2y^2+n^2z^2$$

$$x^2(n^2-1)+2xd-d^2+y^2(n^2-1)+z^2(n^2-1) = 0$$

$$x^2(n^2-1)+y^2(n^2-1)+z^2(n^2-1) = -2xd+d^2$$

$$x^2+y^2+z^2 = \frac{d^2-2xd}{n^2-1}$$

$$\left(x+\frac{2\cancel{x}d}{n^2-1}\right)^2 - \frac{2xd}{n^2-1} - \frac{4d^2}{(n^2-1)^2} + y^2+z^2 = \frac{d^2-2xd}{n^2-1}$$

$$\left(x+\frac{2d}{n^2-1}\right)^2+y^2+z^2 = \frac{4d^2}{(n^2-1)^2}+\frac{d^2-\cancel{2xd}}{n^2-1}+\cancel{\frac{2xd}{n^2-1}}$$

$$= \frac{4d^2+d^4}{(n^2-1)^2} = \frac{d^2(d^2+4)}{(n^2-1)^2}$$

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{|\vec{r}-\vec{r}'|}$$

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\sqrt{1+x} = 1+\frac{x}{2} \quad |x| \ll 1$$

$$\frac{1}{1-x} = 1+x \quad |x| \ll 1$$

$$\int_\Omega \nabla\cdot\vec{F}\,d\Omega = \int_{\partial\Omega}\vec{F}\cdot d\vec{S}$$

Gaussova věta

$\vec{p} = ql\vec{e_y}$ dipolový moment

Př.

dipól

Vypočítej se $\varphi(\vec{r})$ a $E(\vec{r})$

$|\vec{r}| \gg l.$

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{|\vec{r}-\vec{r}_+|}+\frac{-q}{|\vec{r}-\vec{r}_-|}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{|\vec{r}-\vec{r}_-|-|\vec{r}-\vec{r}_+|}{|\vec{r}-\vec{r}_+|\cdot|\vec{r}-\vec{r}_-|}\right) \sim r^2$$

$$\simeq \frac{q}{4\pi\varepsilon_0}\left(\frac{\sqrt{(x)^2+(y+\frac{l}{2})^2}-\sqrt{x^2+(y-\frac{l}{2})^2}}{\sqrt{x^2+(y+\frac{l}{2})^2}\cdot\sqrt{x^2+(y-\frac{l}{2})^2}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\sqrt{1+\left(\frac{y+\frac{l}{2}}{x}\right)^2}-\sqrt{1+\left(\frac{y-\frac{l}{2}}{x}\right)^2}\right)$$

$$\simeq \frac{q\sqrt{x^2+y^2}}{4\pi\varepsilon_0}\left(1+\frac{\left(\frac{y+\frac{l}{2}}{x}\right)^2}{2}-\left(1+\frac{\left(\frac{y-\frac{l}{2}}{x}\right)^2}{2}\right)\right)$$

$$= \frac{q\sqrt{x^2+y^2}}{4\pi\varepsilon_0}\cdot\frac{\cancel{2}+\left(\frac{y+\frac{l}{2}}{x}\right)^2-\cancel{2}-\left(\frac{y-\frac{l}{2}}{x}\right)^2}{2r^2} = \frac{q\sqrt{x^2+y^2}}{4\pi\varepsilon_0}\cdot\frac{\frac{y^2+yl+\frac{l^2}{4}}{x^2}-\frac{y^2-yl+\frac{l^2}{4}}{x^2}}{2r^2}$$

$$= \frac{qx^2}{4\pi\varepsilon_0}\cdot$$

$$\sqrt{x^2+y^2+yl+\frac{l^2}{4}}-\sqrt{x^2+y^2-yl+\frac{l^2}{4}}$$

$$= \frac{q\sqrt{x^2+y^2}}{4\pi\varepsilon_0}\cdot\frac{\sqrt{1+\frac{yl+\frac{l^2}{4}}{x^2+y^2}}-\sqrt{1+\frac{-yl+\frac{l^2}{4}}{x^2+y^2}}}{r^2} = \frac{q r}{4\pi\varepsilon_0}\cdot\frac{\cancel{1}+\frac{\frac{yl+\frac{l^2}{4}}{r}}{2}-\cancel{1}+\frac{+yl-\frac{l^2}{4}}{2r}}{r^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{yl}{r^3} = \frac{qyl}{4\pi\varepsilon_0 r^2}\cdot\frac{\cancel{r}}{\cancel{r}} = \frac{q\vec{e_y}\cdot\vec{r}\cdot l}{4\pi\varepsilon_0 r^3} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$y = \vec{e_y}\cdot\vec{r}$$

$$\vec{E} = -\vec{\nabla}\varphi = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = -\left(\frac{\partial \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}}{\partial x}, \frac{\partial\left(\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right)}{\partial y}, \frac{\partial\left(\frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3}\right)}{\partial z}\right)$$ ③

$r = \sqrt{x^2+y^2+z^2}$ $\qquad \vec{p} = ql\vec{e_y}$

$$\varphi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qly}{r^3} = \frac{qly}{4\pi\varepsilon_0 \cdot (x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$-\nabla\varphi_x = \left(+\frac{+qly \cdot 4\pi\varepsilon_0 \frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}} \cdot 2x}{(4\pi\varepsilon_0 \cdot (x^2+y^2+z^2)^{\frac{3}{2}})^2}\right)$$

$$= \frac{qly\, \cancel{4\pi\varepsilon_0} \cdot 3 \cdot \cancel{r} \cdot x}{4\pi\varepsilon_0 \cdot r^{\cancel{6}\,2}} = \frac{3qlyx}{4\pi\varepsilon_0 r^5}$$

$$-\nabla\varphi_y = \frac{-3qly^2 - qlr^2}{4\pi\varepsilon_0 r^5}$$

$$\boxed{E = \frac{3\vec{p}\cdot\vec{r}\,\vec{r} - \vec{p}r^2}{4\pi\varepsilon_0 r^5}}$$

Př. Skalární potenciál a intenzita dipólu v homogenním poli

$\vec{E} = (0, 0, E_0)$

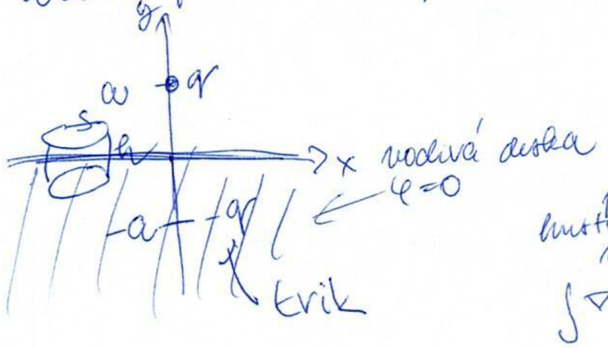
$\vec{p} = (0, 0, p)$

Dokažte, že ekvipotenciální plochou s 0 pot. je koule a určete její poloměr.

$\varphi_0 = -E_0 z$ $\qquad \varphi = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} - E_0 z = \frac{pz}{4\pi\varepsilon_0 r^3} - E_0 z = 0$

$$a = \sqrt{x^2+z^2} = \left(\frac{p}{4\pi\varepsilon_0 E_0}\right)^{\frac{1}{3}}$$

Př. Určete pot. elstat. pole bodového náboje



$\varphi = ?$

$$\varphi(x,y) = \frac{q}{4\pi\varepsilon_0} \cdot \left(\frac{1}{\sqrt{x^2+(y-a)^2}} - \frac{1}{\sqrt{x^2+(y+a)^2}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{(x, a)}{(\sqrt{x^2+(y-a)^2})^3} - \frac{(1, a)}{(\sqrt{x^2+(y+a)^2})^3}\right)$$

Gauss

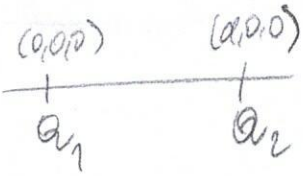
$\nabla \cdot \vec{E} = \rho$

$E_y = \sigma$

plošná hustota náboje

$\int \nabla\cdot\vec{E} = \int\rho = \int E \cdot d\vec{S} \xrightarrow{h\to 0} E_y \cdot S = hS\rho = S\sigma$

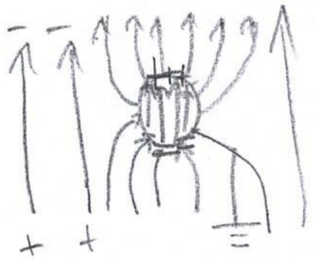
$\Rightarrow E_y = \sigma$

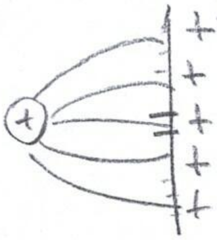


$\frac{Q_2}{Q_1} = n$

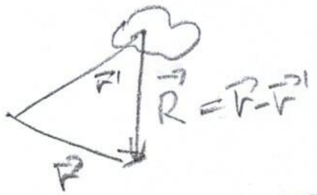
$S = \left(-\frac{d}{n^2-1}, 0, 0\right)$

$a = \frac{dn}{n^2-1}$





homogenně
po uzemnění ⊕ náboj
z desky odteče

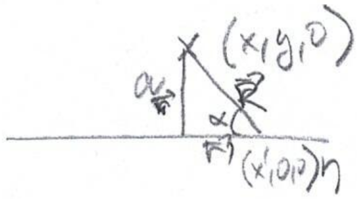


Př. $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^3} \cdot (\vec{r}-\vec{r}')\, dV$

pro bodový náboj: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^3} \cdot \vec{R}$

$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} dV \qquad E = -\nabla\varphi$

σ dS plošná h.
η dl lineární hust.



$\frac{a}{R} = \sin\alpha \Rightarrow R = \frac{a}{\sin\alpha}$ $\qquad \frac{x'}{a} = \cotg\alpha \quad \frac{dx'}{a} = \frac{1}{-\sin^2\alpha}$

nekonečný vodič s lin. hustotou náboje η

$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\eta}{|\vec{r}-\vec{r}'|^3} (\vec{r}-\vec{r}')\, d\vec{r}'$

$E_x = \frac{1}{4\pi\varepsilon_0} \int \frac{\eta}{\cancel{|(x-x')|^3}\,|\vec{r}-\vec{r}'|^3} (x-x')\, dx' = 0$

$E_y = \frac{1}{4\pi\varepsilon_0} \int \frac{\eta}{\cancel{|y|}\,|\vec{r}-\vec{r}'|^3} (y)\, d\cancel{x}' = \frac{1}{4\pi\varepsilon_0} \int_{0} \frac{\eta}{|\vec{R}|^3} \cdot y\, d\cancel{x}'$

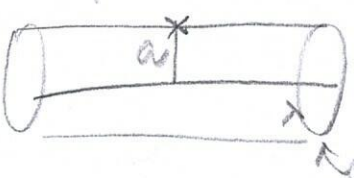
$= \frac{\eta}{4\pi\varepsilon_0} \int \frac{\sin^{\cancel{3}}\alpha}{a^{\cancel{3}}} \cancel{a} \cdot \frac{a}{-\cancel{\sin^2\alpha}}\, d\alpha = \frac{-\eta}{4\pi\varepsilon_0} \int_{\pi}^{0} \frac{\sin\alpha}{a}\, d\alpha$

$= \frac{-\eta}{4\pi\varepsilon_0 a} \int_{\pi} \sin\alpha\, d\alpha = \frac{+\eta}{4\pi\varepsilon_0} \left[+\cos\alpha\right]_{\pi} =$

$= \frac{+\eta}{4\pi\varepsilon_0} (+1+1) = \frac{+\eta}{2\pi\varepsilon_0 a}$

pomocí Gaussovy věty: $\oint \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0} \Longleftrightarrow \oint E\, dS = \frac{Q}{\varepsilon_0}$ pro $\vec{E} \parallel d\vec{S}$

a pro E = konst $\Longleftrightarrow \oint E dS = E \oint dS = ES = \frac{Q}{\varepsilon_0}$



$E = \frac{Q}{\varepsilon_0 S} = \frac{Q}{\varepsilon_0 2\pi r l} = \frac{\lambda \cdot l}{\varepsilon_0 2\pi \lambda l} = \underline{\underline{\frac{\lambda}{2\varepsilon_0 \pi a}}}$

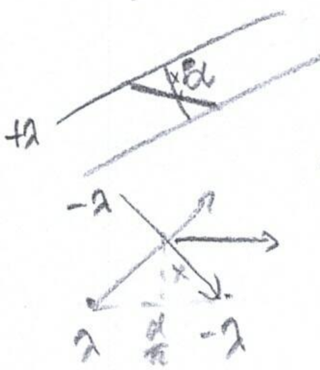
$S = 2\pi r l$

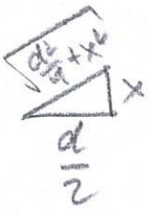
ekvipot. plocha a $\vec{E} \parallel \vec{n}$

Př. (1.22)

$\lambda = \text{konst}$

určete intenzitu v bodě ve výšce x



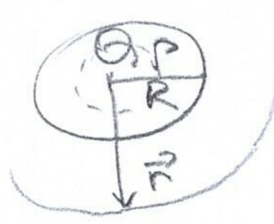


$$E = E_1 + E_2 = \frac{\lambda}{2\varepsilon_0\pi\cdot\sqrt{\frac{d^2}{4}+x^2}}\cdot\frac{\begin{pmatrix}\frac{d}{2}\\ x\end{pmatrix}}{\sqrt{\frac{d^2}{4}+x^2}} + \frac{+\lambda}{2\pi\varepsilon_0\cdot\sqrt{\frac{d^2}{4}+x^2}}\cdot\begin{pmatrix}\frac{d}{2}\\ -x\end{pmatrix}\cdot\frac{1}{\sqrt{\frac{d^2}{4}+x^2}}$$

$$= \frac{\lambda}{2\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)}\begin{pmatrix}\frac{d}{2}\\ x\end{pmatrix} + \frac{+\lambda}{2\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)}\cdot\begin{pmatrix}\frac{d}{2}\\ -x\end{pmatrix}$$

$$= \frac{\lambda}{2\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)}\left(\begin{pmatrix}\frac{d}{2}\\ x\end{pmatrix} + \begin{pmatrix}\frac{d}{2}\\ -x\end{pmatrix}\right) = \frac{\lambda}{2\pi\varepsilon_0\left(\frac{d^2}{4}+x^2\right)}\begin{pmatrix}d\\ 0\end{pmatrix}$$

Př. (1.19) Rovnoměrně nabitá koule
↓ dielektrická



$\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$ ↗

opět je $\vec{n} \parallel \vec{E}$

$Q = \frac{4}{3}\rho\pi r^3$

$r > R$ - vně: ~~$\rho\frac{4}{3}\pi R^3$~~

$ES = \frac{Q}{\varepsilon_0}$

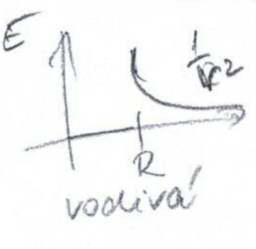
$E = \frac{Q}{S\varepsilon_0} = \frac{Q}{4\pi r^2\varepsilon_0}$

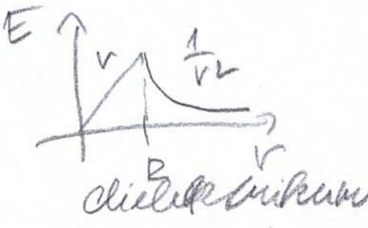
$= \frac{Q}{4\pi\varepsilon_0 r^2}\cdot\frac{\vec{r}}{r}$

$r < R$ - uvnitř:

$q = \frac{Q\,\frac{4}{3}\rho\pi r^3}{\frac{4}{3}\rho\pi R^3} = Q\frac{r^3}{R^3}$

$E = \frac{Q^{\nearrow q}}{S\varepsilon} = \frac{Q\frac{r^3}{R^3}}{4\pi r^2\varepsilon} = \frac{Qr}{4\pi R^3\varepsilon}$





potenciál vně: $r > R$

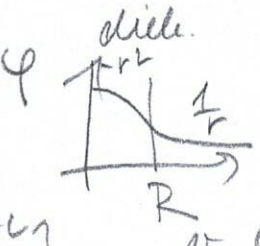
$\vec{E} = -\nabla\varphi$

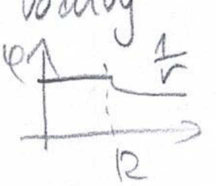
$\vec{E}(r) = -\frac{\partial\varphi}{\partial r}$

$\int\frac{+Q}{4\pi\varepsilon_0 r^2} = \int + \frac{\partial\varphi}{\partial r}$

$\frac{+Q}{4\pi\varepsilon_0}\int\frac{1}{r^2}\,dr = \int d\varphi$

$\varphi \overset{c}{=} \frac{Q}{4\pi\varepsilon_0}\, r^{-1} \overset{c}{=} \frac{Q}{4\pi\varepsilon_0 r} \overset{c}{=} \frac{-Q}{4\pi\varepsilon_0 r} + \underset{\downarrow 0}{c_1}$ v ∞ 0 pot.





potenciál uvnitř $r < R$:

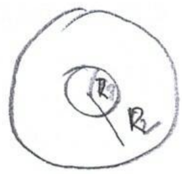
$\vec{E}(r) = -\frac{\partial\varphi}{\partial r}$

$\int\frac{Qr}{4\pi R^3\varepsilon} = \int -\frac{\partial\varphi}{\partial r}$

$\varphi = -\frac{Q}{4\pi R^3\varepsilon}\int r\,dr \overset{c}{=} -\frac{Q}{4\pi R^3\varepsilon}\cdot\frac{r^2}{2} + c_2$

↓ pro $r = R$ $= \frac{-Q}{4\pi\varepsilon_0 r}$

protože potenciál je vždy spojitý!



$$\varphi_1 = A_{11}Q_1 + A_{12}Q_2$$

$$\varphi_2 = A_{21}Q_1 + A_{22}Q_2$$

$\varphi = AQ$ A ji nějaká konst. influenční koef.

$C\varphi = \underbrace{CA}_{I}Q$

$C\varphi = Q$

$$Q_1 = C_{11}\varphi_1 + C_{12}\varphi_2$$

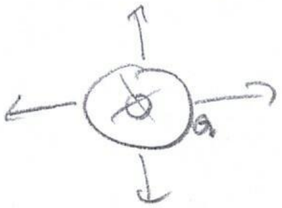
$$Q = C_{21}\varphi_1 + C_{22}\varphi_2$$

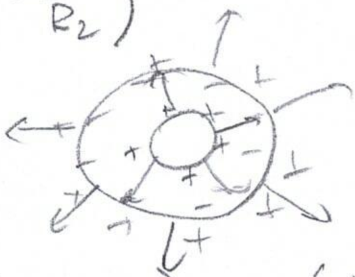
kapacitní koef.

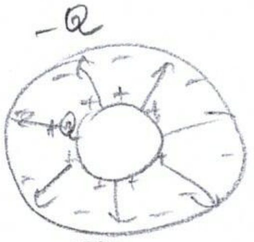
$Q = Q_1 - Q_2$

$$\varphi_1 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\varphi_2 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_2}\right)$$







$$\left(\begin{array}{cc|cc} \frac{1}{R_1} & \frac{1}{R_2} & 1 & 0 \\ \frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & \frac{R_1R_2}{R_2-R_1} & -\frac{R_1R_2}{R_2-R_1} \\ 0 & 1 & -\frac{R_1R_2}{R_2-R_1} & \frac{R_2^2}{R_2-R_1} \end{array}\right)$$

influenční k.

kapacitní k.

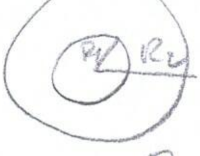
$C_{12} = C_{21}$ symetrie

$C_{11} = -C_{12}$ podm. kondenzátoru

KAPACITA

$$C = \frac{Q}{U} \qquad U = \varphi_2 - \varphi_1$$

Př. 1.2.6. kondenzátor kapacita pomocí kap. a infl. koeficientů



$$U = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{Q}{U} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = 4\pi\varepsilon_0 \frac{R_2R_1}{R_2-R_1}$$

$$U = \varphi_A - \varphi_B = Q\left(\frac{C_{22} - C_{12} - C_{11} + C_{21}}{C_{11}C_{22} - C_{12}C_{21}}\right)$$

Př. 1.2.7.

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} \qquad S = 2\pi r l$$

$$SE = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{2\pi r l\varepsilon_0} = \frac{\sigma \cdot 2\pi R l}{2\pi r\varepsilon_0 l} = \frac{\sigma R}{r\varepsilon_0}$$

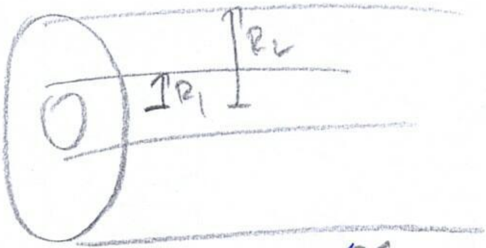
$E = -\nabla\varphi$ ~~φbu~~ $\int E = \int -\frac{\partial\varphi}{\partial r}$

$$\int \frac{\sigma R}{r\varepsilon_0}\,dr = -\int d\varphi$$

$$\varphi = -\frac{\sigma R}{\varepsilon_0}\ln|r| + c = \frac{\sigma R}{\varepsilon_0}\ln\frac{1}{r} + c$$

nemůže být 0 pot. v ∞ → bude na elektrodě

$$\Rightarrow c = \frac{\sigma R}{\varepsilon_0}\ln R \rightarrow \varphi = \frac{\sigma R}{\varepsilon_0}\ln\left(\frac{R}{r}\right)$$



$$\varphi_1 = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_1} + \frac{\sigma_2 R_2}{\varepsilon_0}\ln\frac{R_1}{R_2}$$

$$\varphi_2 = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_2} + \frac{\sigma_2 R_2}{\varepsilon_0}\ln\frac{R_1}{R_2}$$ integr. konst.

$$U = \varphi_2 - \varphi_1 = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_2}$$

$$C = \frac{Q}{U} = \frac{\sigma_1 R_1 2\pi l}{-\frac{\sigma_1 R_1}{\varepsilon_0}\ln\frac{R_1}{R_2}} = \frac{2\pi\varepsilon_0 l}{-\ln\left(\frac{R_1}{R_2}\right)}$$

$$ES = \frac{Q}{\varepsilon_0} \qquad E\, 2\pi r l = \frac{Q}{\varepsilon_0} \qquad E = \frac{Q}{2\pi\varepsilon_0 l}\cdot\frac{1}{r} \Rightarrow \varphi = -K\int\frac{1}{r} = K\cdot\log\frac{1}{r} + c$$

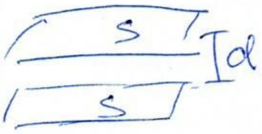
$$= K\log\frac{r_0}{r}$$

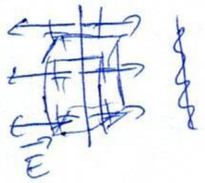
$$\varphi_1 - \varphi_2 = \frac{Q}{2\pi\varepsilon_0 l}\left(\ln\frac{R_1}{R_1} - \ln\frac{R_1}{R_2}\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\log\frac{R_2}{R_1}$$

$$\boxed{-\int_A^B \vec{E}\cdot d\vec{l} = U} = -\int_{R_2}^{R_1}\frac{Q}{2\pi\varepsilon_0 l}\cdot\frac{1}{r}\,dr = -\frac{Q}{2\pi\varepsilon_0 l}\left[\log R_1 - \log R_2\right] = \frac{Q}{2\pi\varepsilon_0 l}\log\frac{R_2}{R_1}.$$

Př. (1.2.3) Kapacita deskového kond.



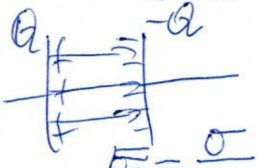


$$\oint\vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

$$2E\cdot S = \frac{Q}{\varepsilon_0} \qquad C = \frac{Q}{U}$$

$$= \frac{\sigma S}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

$$\varphi = -\int\frac{\sigma}{2\varepsilon_0}\,dx \overset{c}{=} -\frac{\sigma}{2\varepsilon_0}x$$



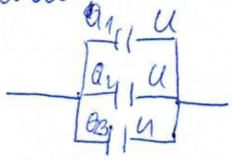
$$E = \frac{\sigma}{\varepsilon_0} \qquad \left(2\times\frac{\sigma}{2\varepsilon_0}\right)$$

$$U = -\int_d^0 \frac{\sigma}{\varepsilon_0}\,dx = -\frac{\sigma}{\varepsilon_0}(0 - d)$$

$$= \frac{d\sigma}{\varepsilon_0}$$

$$C = \frac{Q}{U} = \frac{\cancel{\sigma} S}{\frac{d\cancel{\sigma}}{\varepsilon_0}} = \frac{\varepsilon_0 S}{d}$$

Př. Kapacita při sériovém a paralelním zapojení



$$C_1 = \frac{Q_1}{U} \quad C_2 = \frac{Q_2}{U} \quad C_3 = \frac{Q_3}{U}$$

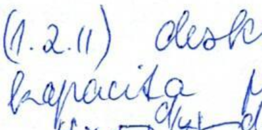
$$Q = Q_1 + Q_2 + Q_3$$

$$C = \frac{Q}{U} = \frac{Q_1 + Q_2 + Q_3}{U} = \frac{Q_1}{U} + \frac{Q_2}{U} + \frac{Q_3}{U} = C_1 + C_2 + C_3.$$

$$U = U_1 + U_2 + U_3 \qquad Q = Q_1 = Q_2 = Q_3$$

$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C} \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Př. (1.2.11) deskový kondenzátor s kapacitou C. Jaká bude kapacita po vložení plechu?

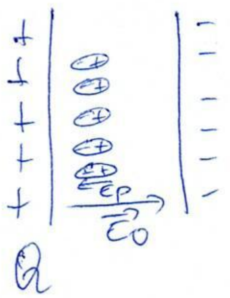


$d - x - \frac{d}{4}$ dva sériově zapojené kondenzátory

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{x}{S\varepsilon_0} + \frac{d - x - \frac{d}{4}}{S\varepsilon_0} =$$

$$= \frac{d - x - \frac{d}{4} + x}{S\varepsilon_0} = \frac{\frac{3}{4}d}{S\varepsilon_0} \Rightarrow C = \frac{4}{3}\frac{S\varepsilon_0}{d}$$

Př.



$$\vec{E} = \vec{E}_0 \longrightarrow \vec{E} = \vec{E}_0 + \vec{E}_p$$

v x: $E = E_0 - E_p$

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$$

$$\varepsilon_0 E = \sigma - \sigma_p \qquad \sigma_p = \vec{P}\cdot\vec{n}$$

$$\varepsilon_0 E = \sigma - P \qquad P = \chi \varepsilon_0 E$$

$$\varepsilon_0 E = \sigma - \chi \varepsilon_0 E$$

$$\varepsilon_0 E \underbrace{(1+\chi)}_{\varepsilon_r} = \sigma \longrightarrow E \underbrace{\varepsilon_0 \varepsilon_r}_{\varepsilon} = \sigma$$

$$\underbrace{E\varepsilon}_{} = \sigma$$

$$D = \sigma \quad /\cdot S$$

vektor el. indukce

$$\oint \vec{D}\cdot d\vec{S} = Q$$

Př. (1.3.1) Deskový kondenzátor o kap. C_0, ploše S, nábojem Q.
Přidali jsme diel. a $C_0 \to C$. Vypočtěte $\vec{E}$, σ_p $\quad C = \frac{\varepsilon_0 S}{d}$
kond. není připojen ke zdroji $\Rightarrow Q$ = konst

$\vec{E}$ před vložením diel.

$$Ed = U = \frac{Q}{C} \Rightarrow E_0 = \frac{Q}{C_0 d} = \frac{Q}{\frac{\varepsilon_0 S}{d}\cdot d} = \frac{Q}{\varepsilon_0 S} = \frac{\sigma S}{\varepsilon_0 S} = \frac{\sigma}{\varepsilon_0}$$

po vložení:

$$E = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \Rightarrow \frac{E}{E_0} = \frac{\frac{\cancel{\sigma}}{\cancel{\varepsilon_0}\varepsilon_r}}{\frac{\cancel{\sigma}}{\cancel{\varepsilon_0}}} = \frac{1}{\varepsilon_r}$$

σ_p:

$$-\sigma_p = \vec{P}\cdot\vec{n} = P = \chi \varepsilon_0 E \qquad \varepsilon_r = \chi + 1$$

$$\chi = \varepsilon_r - 1$$

$$= (\varepsilon_r - 1)\varepsilon_0 E = (\varepsilon_r - 1)\varepsilon_0 \frac{E_0}{\varepsilon_r}$$

$$= (\varepsilon_r - 1)\cancel{\varepsilon_0}\cdot\frac{\sigma}{\cancel{\varepsilon_0}\varepsilon_r} = \frac{\sigma(\varepsilon_r - 1)}{\varepsilon_r}$$

U:

$$U = E\cdot d = \overset{E_0}{\boxed{\frac{\sigma}{\varepsilon_0 \varepsilon_r}}}\, d = \frac{E_0}{\varepsilon_r}\, d$$

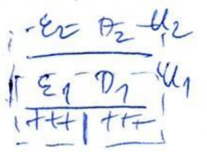
při U=konst:

$$\sigma = \sigma_0 \varepsilon_r$$

σ_p:

$$\sigma_p = P = \chi \varepsilon_0 E = (\varepsilon_r - 1)\varepsilon_0 \cdot E_0$$

Př. (1.3.4)



$$U = U_1 + U_2 \qquad DS = Q$$

$$D = E\varepsilon$$

$$D = D_1 = D_2$$

$$\varepsilon E = \varepsilon_1 E_1 = \varepsilon_2 E_2 \qquad \varepsilon_2 > \varepsilon_1 : E_2 < E_1$$

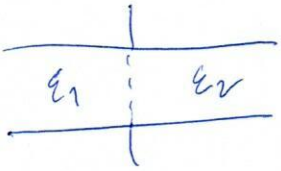
$$D = \frac{Q}{S} \qquad D = \varepsilon E \qquad U = E_1 d + E_2 d = \cancel{\frac{\sigma}{\varepsilon_1} d + \frac{\sigma}{\varepsilon_2} d}$$

$$= \frac{D}{\varepsilon_1} d + \frac{D}{\varepsilon_2} d = D\left(\frac{d}{\varepsilon_1} + \frac{d}{\varepsilon_2}\right) = \frac{Q}{S}\left(\frac{d}{\varepsilon_1} + \frac{d}{\varepsilon_2}\right)$$

$$\cancel{= Q\left(\frac{d}{S\varepsilon_1} + \frac{d}{S\varepsilon_2}\right) = \frac{Qd}{S\varepsilon_1}} \; Q\left(\frac{d}{S\varepsilon_1} + \frac{d}{S\varepsilon_2}\right)$$

$$\frac{1}{C} = \frac{U}{Q} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{S\varepsilon_1} + \frac{d}{S\varepsilon_2}$$

Př.



$U_1 = U_2 = U = Ed$

$CU = Q = Q_1 + Q_2 = D_1\frac{S}{2} + D_2\frac{S}{2} = \frac{\varepsilon_1 E S}{2} + \frac{\varepsilon_2 E S}{2}$

$= \frac{\overset{U}{\boxed{Ed}}\,\overset{C_1}{\varepsilon_1 S}}{\boxed{2d}} + \frac{\overset{U}{\boxed{Ed}}\,\overset{C_2}{\varepsilon_2 S}}{\boxed{2d}} \Rightarrow C = C_1 + C_2$

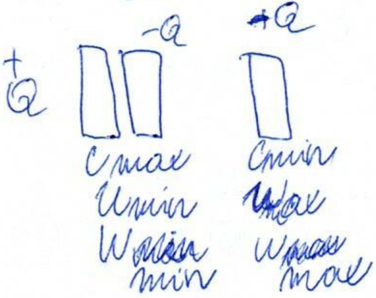
Př (1.4.1)

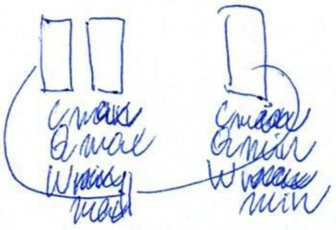
Energie kondenzátoru

$$W = \int_0^Q U\,dQ = \frac{1}{C}\int_0^Q Q\,dQ = \frac{1\,Q^2}{2C} = \frac{1}{2}QU = \frac{1}{2}CU^2$$

$C = \frac{\varepsilon S}{d}$ $\quad C = \frac{Q}{U}$ $\quad D = \varepsilon E$ $\quad Q = DS$

$F = -\text{grad}\,W$





→ zdroj musí vykonat větší práci na donesení náboje

Př. (1.4.1) otočný kondenzátor

min. kapacita C_0. Jakou W vykonáme při změně $C_{max} \to C_0$, jestliže na elektrodách je U=konst?

$W_0 = \frac{1}{2}C_0U^2$ $\qquad W_{max} = \frac{1}{2}C_{max}U^2$

$\Rightarrow \delta W = \frac{1}{2}(C_{max} - C_0)U^2$

Jakou práci vykonáme, jestliže byl nabit na napětí U a při otáčení zdroj odpojíme?

$W_0 = \frac{1}{2}\frac{Q^2}{C_0}$ $\qquad W_{max} = \frac{1}{2}\frac{Q^2}{C_{max}}$

$\delta W = \frac{1}{2}\left(\frac{Q^2}{C_0} - \frac{Q^2}{C_{max}}\right) = \frac{Q^2}{2}\left(\frac{1}{C_0} - \frac{1}{C_m}\right) = \frac{Q^2}{2}\frac{C_m - C_0}{C_0C_m}$

Př. Jaká síla působí na elektrony desk. kond. nabitého na nap. U, plocha je S, vzd. x.

$C = \frac{\varepsilon S}{x}$ $\qquad F = -\text{grad}\,W$

$W = \frac{1}{2}CU^2 = \frac{1}{2}\frac{\varepsilon S}{x}U^2$

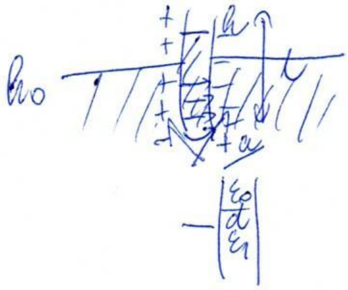
$F_x = -\frac{1}{2}\frac{\varepsilon S}{x}U^2\cdot\frac{\partial}{\partial x} = +\frac{1}{2}\varepsilon S U^2\cdot\left(+\frac{1}{x^2}\right) = \frac{1}{2}\frac{\varepsilon S U^2}{x^2}$

$F_y = 0$

$F_z = 0$

$\underline{\vec{F} = \left(\frac{1}{2}\frac{\varepsilon S U^2}{x^2}, 0, 0\right)}$

Př. (1.4.6) kond. ponořený do kapaliny
kap. vystoupá! proč?



$$mgh = \frac{1}{2} CU^2$$

~~$h = \frac{1}{2}\frac{CU^2}{mg} = \frac{1}{2}\frac{CU^2}{\rho V g} = \frac{CU^2}{\rho 2 S g}$~~

$$W = \frac{1}{2}(C_1 + C_2)U^2 = \frac{1}{2}\left(\frac{\varepsilon_1 a h}{d} + \frac{\varepsilon_0 (l-h) a}{d}\right)U^2$$

$$C = \frac{\varepsilon S}{d}$$

$$= \frac{a}{2d}\left(\varepsilon_1 h - \varepsilon_0(l-h)\right)U^2 = \frac{a}{2d}\left(l(\varepsilon_1-\varepsilon_0)+\varepsilon_0 h\right)U^2$$

~~$mgh = \frac{a}{2d}\left([illegible] + \varepsilon_0 h\right)$~~

zderivujeme → grad → F

$mgh = F$

Pozn. bod. náboj: $W = QU$

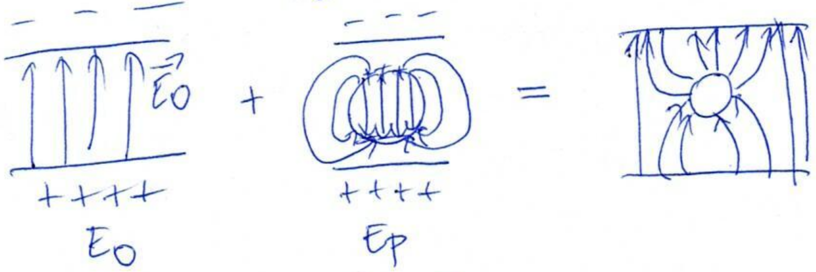
dipól: $W = -\vec{p}_V \cdot \vec{E}_0$ ← vnější pole

$\vec{E} = \vec{E}_0 + \vec{E}_P$
↑výsl. ↖ pole zpol. diel.

dielektr. tvrdé: $W = -\int \vec{P}_0 \vec{E}_0 \, dV$

diel. měkké: $W = -\int \vec{P}_m \cdot \vec{E}_0 \, dV + \frac{1}{2}\int \vec{P}_m \cdot \vec{E}_0 \, dV = -\frac{1}{2}\int \vec{P}_m \cdot \vec{E}_0 \, dV$

Př. do homog. pole $\vec{E}_0$ byla vložena koule o pol. R. Spočítejte výsl. energii (již měkkého diel.) koule.



$$W = -\frac{1}{2}\int \vec{P}_m \cdot \vec{E}_0 \, dV$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma}{R^2} \, dS$$

$$E_p = \frac{-P}{3\varepsilon_0}$$

$$\vec{P} = \varepsilon_0 \varkappa \vec{E}$$

$$\vec{E} = \vec{E}_0 + \vec{E}_p$$

$$\sigma_p = \vec{P} \cdot \vec{n} = P\cos\vartheta$$

$$E = E_0 - \frac{P}{3\varepsilon_0} = E_0 - \frac{\varepsilon_0 \varkappa E}{3\varepsilon_0}$$

$$E\left(1 - \frac{\varepsilon_0 \varkappa}{3\varepsilon_0}\right) = E_0 \Rightarrow E = \frac{E_0}{1 - \frac{\cancel{\varepsilon_0}\varkappa}{3\cancel{\varepsilon_0}}} = \frac{E_0}{\frac{3-\varkappa}{3}} = \frac{3E_0}{3-\varkappa} =$$

$$= \frac{3E_0}{3-(\varepsilon_r - 1)}$$

$$P = \frac{(\varepsilon - \varepsilon_0) 3\varepsilon_0 E_0}{2\varepsilon_0 + \varepsilon}$$

$$W = -\frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{\cancel{3}} \pi r^3 \cdot \frac{(\varepsilon-\varepsilon_0)\cancel{3}\varepsilon_0 E_0}{2\varepsilon_0 + \varepsilon} \cdot E_0 = \frac{(\varepsilon_0 - \varepsilon)\varepsilon_0 E_0^2}{2\varepsilon_0 + \varepsilon}$$

ELEKTRICKÝ PROUD, OBVODY.

Př. železný drát   Jaká je V-A charakteristika variátoru?
R závisí lineárně na teplotě.

$$R_t = R_0\left(1+\alpha t+\beta t^2+\dots\right)$$

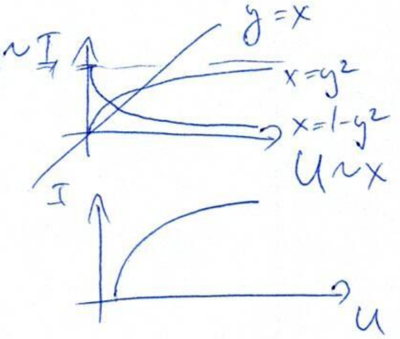
$$R=\frac{U}{I}=R_0\left(1+\alpha\Delta t\right)=R_0\left(1+\alpha g UI\right)$$

$$U\left(1-R_0I^2\alpha g\right)=R_0I$$

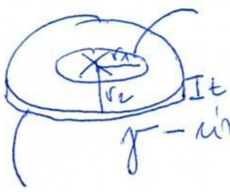
$$U=\frac{R_0I}{1+R_0I^2\alpha g}$$

$$\sqrt{R_0\alpha g}\,I=y$$

$$U=\frac{R_0I}{1+y^2}\cdot\frac{\sqrt{\frac{\alpha g}{R_0}}}{\sqrt{\frac{\alpha g}{R_0}}}=\sqrt{\frac{\alpha g}{R_0}}\,U=x=\frac{y}{1+y^2}$$



Př. diska   Stanovte odpor mezikruží!



γ – úměrná vodivost

$$R=\frac{U}{I}$$

$$E=\frac{\sigma}{\varepsilon_0}\cdot\frac{r_1}{r}$$ – už víme

$$U=\frac{\sigma_2 r_2}{\varepsilon_0}\ln\frac{r_2}{r_1}-\cancel{\frac{\sigma_1 r_1}{\varepsilon_0}\ln\frac{r_1}{r_1}}=\frac{\sigma_2 r_2}{\varepsilon_0}\ln\frac{r_2}{r_1}$$

$$I=\int\vec{i}\cdot d\vec{S}=\gamma\int\vec{E}d\vec{S}=\gamma\frac{\sigma_1}{\varepsilon_0}\cdot\frac{r_1}{\cancel{r}}2\pi\cancel{r}t=$$

$$\vec{i}=\gamma\vec{E}$$

$$\sigma_1 r_1=Q=\sigma_2 r_2$$

$$R=\frac{U}{I}=\frac{\frac{\cancel{\sigma_1 r_1}}{\cancel{\varepsilon_0}}\ln\frac{r_2}{r_1}}{\gamma\frac{\cancel{\sigma_1}}{\cancel{\varepsilon_0}}\cancel{r_1}\,2\pi t}=\frac{\ln\frac{r_2}{r_1}}{\gamma 2\pi t}$$

$$\underbrace{i\Delta S}_{I}=\underbrace{\frac{\gamma\Delta S}{\Delta r}}_{\frac{1}{R}}\underbrace{E\Delta r}_{U}$$

jinak: $dR=\frac{dr}{\gamma S(r)}\Rightarrow R=\frac{1}{\gamma}\int_{r_1}^{r_2}\frac{dr}{2\pi rt}=\frac{1}{\gamma 2\pi t}\underbrace{\int_{r_1}^{r_2}\frac{dr}{r}}_{\ln\frac{r_2}{r_1}}$

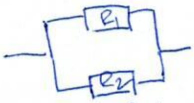
Př.

I

U

sériové odpory $U=U_1+U_2$ $I=I_1=I_2$

$$R=\frac{U}{I}=\frac{U_1+U_2}{I}=R_1+R_2$$



$U=U_1=U_2$

$I=I_1+I_2$

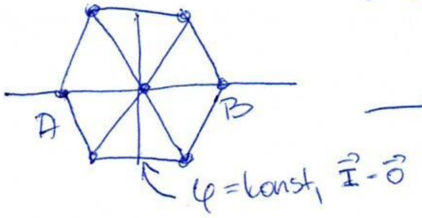
paralelní odpory

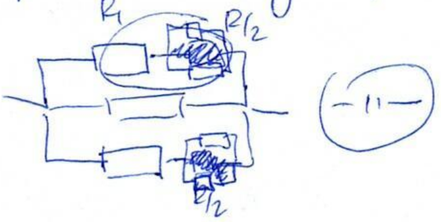
$$R=\frac{U}{I}=\frac{U}{I_1+I_2}$$

$$R^{-1}=\frac{I_1+I_2}{U}=\frac{I_1}{U}+\frac{I_2}{U}=R_1^{-1}+R_2^{-1}$$

Př.
5.1.4.

Určete odpor mezi body A, B.





$R_1 = \frac{4R}{3} = \frac{3}{R}+R$

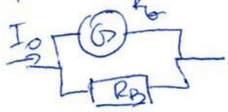
$\frac{1}{\frac{R_1}{2}} + \frac{1}{R} = \frac{3}{R}$

$\frac{2}{R_V} = \frac{6}{4R} + \frac{1}{R} = \frac{5}{2}R$

$\frac{1}{\cancel{2}}R_V = \frac{2}{5}R$ $\underline{R_V = \frac{4}{5}R}$

Př.
5.1.11.

Ke Galvanometru s vnitřním odporem R_G je přip. bočník s odp. R_B, který 10x snižuje citlivost Galvanometru. Jaký ~~odpor~~ obvod musíme připojit, aby to fungovalo?



R=? aby citlivost snížil 10x?

$I_G = \frac{1}{10}I_0$ $U = R_0 I_0 = R_1 I_1$

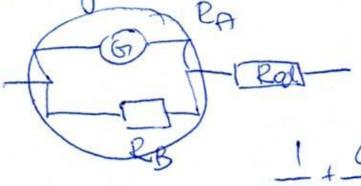
$I_0 = I_B + I_G \Rightarrow I_B = \frac{9}{10}I_0$

~~$R_B = \frac{U_B}{I_B} = \frac{U}{\frac{9}{10}I_0} \Rightarrow \frac{10}{9}R_0$~~ (← pravda)

$I_B = 9 I_G$ $R = \frac{U}{I}$

$\frac{\cancel{U_B}}{R_B} = 9\frac{\cancel{U}}{R_G} \Rightarrow R_B = \frac{1}{9}R_G$

Připojíme ještě odpor tak, aby R bylo stejné jako R_G.



$\frac{1}{R_G} + \frac{1}{R_B} = \frac{1}{R_A}$

$\frac{1}{R_G} + \frac{9}{R_G} = \frac{1}{R_A} \Longleftrightarrow \frac{1}{R_G} + \frac{1}{\frac{1}{9}R_G} = \frac{1}{R_A}$

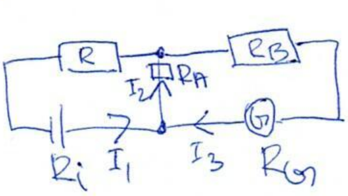
$\frac{10}{R_G} = \frac{1}{R_A} \Rightarrow R_A = \overset{\frac{1}{10}}{\cancel{\frac{9}{10}}}R_G$

$R_{od} + \overset{\frac{1}{10}}{\cancel{\frac{9}{10}}}R_G = R_G$

$R_{od} = \frac{9}{10}R_G.$

Kirchhoffovy zákony

Př. 5.1.16



g uzlů, poč. nezávislých je g-1

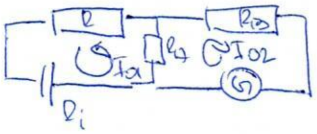
$\sum I_i = 0$ $I_1 + I_3 = I_2$

n větví, l smyček, poč. nezávislých je n-(g-1)

$\sum R_i I_i = \sum \varepsilon_i$ ← zdroje napětí

$I_1(R+R_i) + I_2 R_A = \varepsilon$

$I_3(R_B+R_G) + I_2 R_A = 0$

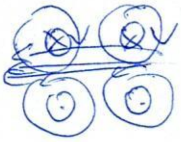


$I_{01}(R+R_i+R_A) + I_{02}R_A = \varepsilon$

$I_{02}(R_A+R_B+R_G) + I_{01}R_A = 0$

Př. 3.1.6. Užitím Ampérova z. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{celk}$ určete vztah mezi intenzitou a mag. polem solenoidu.

cívka, kterou teče proud



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{celk} = \int_{l_1} B\,dl + \underbrace{\overset{0}{\overset{\parallel}{\int_{l_2} \vec{B}\cdot d\vec{l}}}}_{B\perp l} + \underbrace{\overset{0}{\overset{\parallel}{\int_{l_3} \vec{B}\cdot d\vec{l}}}}_{B\perp l}$$

až do ∞ ⇒ spodní str. má 0 příspěvek

$$+ \overset{0}{\overset{\parallel}{\int_{l_4} \vec{B}\cdot d\vec{l}}} = \int_{l_1} B\,dl$$

l_4 v ∞ → $\vec{B} = \vec{0}$

$$\Longrightarrow B\cancel{l_1} = \mu_0 N I$$
$$= \mu_0 n \cancel{l_1} I$$
$$B = \mu_0 n I$$

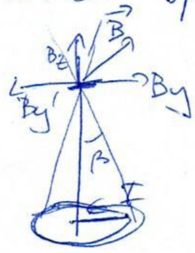
$n = \frac{N}{l_1}$ hustota smyček

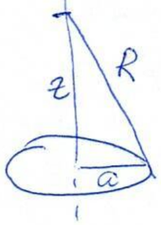
vekt. potenciál

$$\vec{A} = \cancel{\mu}\frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}}{R} \qquad \vec{B} = \mathrm{rot}\,\vec{A}$$

(obdobně jako $\varphi = \frac{1}{4\pi\varepsilon_0}\int\frac{Q}{R}$ $\qquad \vec{E} = -\nabla\varphi$)

Př. 3.1.13. Spočítejte mag. pole na ose závitu





$R^2 = z^2 + a^2$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\vec{R}}{R^3}$$

my máme jen B ve směru osy z

$$\vec{B} = \frac{\mu_0 I}{\cancel{\;}4\pi}\int\frac{dl\cdot R\cdot\sin\alpha}{R^{\cancel{3}2}}$$

$$= \frac{\mu_0 I}{4\pi}\cdot\frac{1}{R^2}\cdot\sin\alpha\cdot\int dl = \frac{\mu_0 I}{\cancel{4\pi}2}\cdot\frac{1}{R^2}\cdot 2\cancel{\pi}a\cdot 1 = \frac{\mu_0 I}{2R^2}\cdot a$$

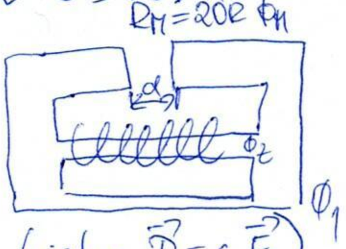
$\sin\alpha = 1$

$\alpha = 90°$

$$B_z = B\cos\beta = B\cdot\frac{a}{R} = \frac{\mu_0 I a^2}{2R^3}$$

MAGNETICKÉ OBVODY

Př. 3.2.2.



Najděte intenzitu mag. pole v mezeře.

$\vec{B} = \mu \vec{H}$ (jako $\vec{D} = \varepsilon \vec{E}$)

$\oint \vec{B} \cdot d\vec{l} = \mu I$

$\oint \vec{H} \cdot d\vec{l} = I$

zdroj magnetomotorického napětí

$\mathcal{F} = \oint \vec{H}\, d\vec{l} = NI$ ← poč. závitů

$\Phi = \int \vec{B} \cdot d\vec{S}$

mag. tok

$R_M = \int \frac{dl}{\mu S}$

mag. odpor

jako

$\mathcal{E}$

I

R

$U = \int \vec{E}\, d\vec{l}$ | $U = RI$

$U_m = \int \vec{H}\, d\vec{l}$ | $U_m = R\Phi$

$\vec{B}$ se při $\vec{S} = \text{konst}$ nemění, mění se $\vec{H}$

platí Kir

$\Sigma = RI$

$\mathcal{F} = R_M \Phi = NI$

~~$\Phi_Z R + \Phi_M 21R$~~

$$\Phi_Z = \Phi_M + \Phi_1$$

$$\Phi_1 R + \Phi_Z R = \mathcal{F}$$

$$\Phi_Z R + \underbrace{(20R + R)}_{R_M} \Phi_M = \mathcal{F}$$

$$\Phi_Z R + \Phi_M 21R = \Phi_1 R + \Phi_Z R$$

$$\Phi_M = \frac{\Phi_1 R}{21R} = \frac{\Phi_1}{21}$$

$$\Phi_M = \frac{\mathcal{F} - \Phi_Z R}{21R} = \frac{\mathcal{F} - (\Phi_M + \Phi_1)R}{21R}$$

$$0 = \mathcal{F} - 21R\Phi_M - \Phi_M R - \Phi_1 R$$

$$\Phi_M = \frac{\mathcal{F} - \Phi_1 R}{22R}$$

$$\Phi_M = \frac{\mathcal{F} - 21\Phi_M R}{22R}$$

$$22R\Phi_M = \mathcal{F} - 21\Phi_M R$$

$$43\Phi_M R = \mathcal{F}$$

$$\Phi_M = \frac{\mathcal{F}}{43R}$$

$$\underset{\varepsilon}{H_M d} = \overset{R}{\boxed{20R}}\boxed{\frac{\mathcal{F}}{43R}}I$$

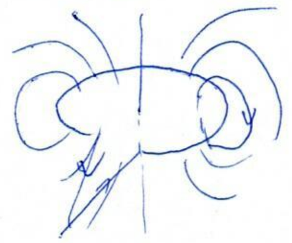
$$\int \vec{H} \cdot d\vec{l} = \int_d H\, dl = Hd$$

$$H_M = \frac{20\mathcal{F}}{43d}$$

jinak: $\Phi = \int \vec{B}\, d\vec{S} = \int B\, dS = BS = \frac{\mathcal{F}}{43R}$

$R_M = \frac{d}{\mu S}$ $\mu HS = \frac{\mathcal{F}}{43R} \rightarrow H = \frac{\mathcal{F}\mu S}{\mu S \cdot 43 \cdot d}$

poznámka k minulému cv.



znali jsme směr, proto jsme místo

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\vec{R}}{R^3}$$

počítali

$$B = \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\alpha}{R^2}$$

mohli jsme počítat

$$B_z = \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\alpha}{R^2}\frac{a}{\sqrt{a^2+z^2}}$$ ← průmět na osu z

což vyjde stejně jako to, co jsme počítali minule.

Další př. se dal spočítat z $\int H d\vec{l} = \phi_2\, 20 R_m$

$H = \frac{20 F}{43 d}$ $\vec{B} = \mu\vec{H}$

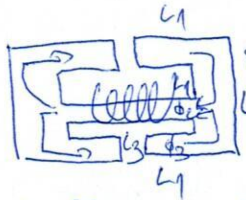
nebo $\oint B d\vec{S} = BS$

$$\mu H S = \frac{F}{43 R_m}$$

$$R_m = \frac{R_M}{20} = \frac{d}{\mu_0 S\cdot 20}$$

$$H = \frac{F \cancel{\mu_0}\cancel{S} 20}{43 d \cancel{\mu_0}\cancel{S}} = \frac{20 F}{43 d}$$

Př. 3.2.5.



Kolik A musí být v závitech, aby v mezerách bylo pole B. Známe L_i, S.

$$F = \oint \vec{H}\cdot d\vec{l} = NI$$

$$R_m = \int \frac{dl}{\mu S} = \frac{l}{\mu S}$$

$\Phi_2 = \phi_1 + \phi_3$ $\phi_1 = \phi_2$

$$NI = \phi_1\left(\frac{L_1}{\mu S} + \frac{L_2}{\mu S} + \frac{L_3}{\mu_0 S}\right) + 2\phi_1\frac{L_1}{\mu S}$$

$$NI = 3\phi_1\frac{L_1}{\mu S} + \phi_1\frac{L_2}{\mu S} + \phi_1\frac{L_3}{\mu_0 S}$$ $\phi_1 = BS$

$$NI = \cancel{\phi} B\cancel{S}\left(3\frac{L_1}{\mu\cancel{S}} + \frac{L_2}{\mu\cancel{S}} + \frac{L_3}{\mu_0\cancel{S}}\right) \Rightarrow I = \frac{B}{N}\left(3\frac{L_1}{\mu} + \frac{L_2}{\mu} + \frac{L_3}{\mu_0}\right).$$

Př. 3.3.3

$$\mathcal{E} = -\frac{\partial\phi}{\partial t}$$

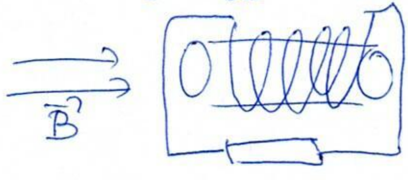
↳ elektromotorické napětí

$\phi = \int\vec{B}\cdot d\vec{S}$
- mění se B
- mění se S
- mění se ϕ

$\phi = LI$ jako $Q = CU$

tok ↗ ↖ induktivnost

trubka z izolačního materiálu



N závitů

$\vec{B}$ homog. mag. pole

známe D - průměr cívky

máme spočítat proudový impulz, ke kterému dojde při otočení o 180°

$\phi_1 = BS = B\cdot\pi\left(\frac{D}{2}\right)^2 N$

$\phi_2 = -BS = B\pi\left(\frac{D}{2}\right)^2 N$

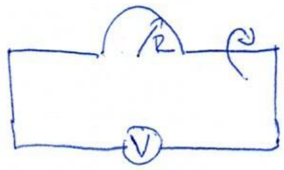
pr. impulz je $\int^{\infty} I dt$ $\cancel{I}\ -\mathcal{E}\Delta t = \Delta\phi$

napěťový impulz je tedy $\phi_2 - \phi_1 = -\pi B\frac{D^2}{2}N$

Př. 3.3.5. pevný drát se otáčí v homog. mag. poli s frekvencí f.

Voltmetr ukazuje napětí U.

Jaká je mag. indukce B?



$\omega = 2\pi f$

$$\Phi = \int \vec{B}\cdot d\vec{S} = \int B\cos\varphi\, dS = \int B\cos\omega t \cdot dS$$

$$= \int B\cos(2\pi f t)\, dS = B\cos(2\pi f t) S$$

$$= B\cos(2\pi f t)\frac{\pi R^2}{2}$$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = +\frac{B\pi R^2}{\cancel{2}}\cdot\sin(2\pi f t)\cdot\cancel{2}\pi f = B\pi^2R^2 f\sin(2\pi f t)$$

Voltmetr změří časovou střední hodnotu (nestíhá se měnit)

$$U^2_{2ft} = \frac{1}{T}\int_0^T u^2\, dt \qquad \int_0^T \sin^2 t + \int_0^T \cos^2 t = \int_0^T 1$$

$$A + A = T$$

$$\int_0^T \sin^2 dt = A - U^2_{2ft} = \frac{T}{2} \qquad A = \frac{T}{2}$$

$$U_{2ft} = \frac{T}{\sqrt{2}} \qquad U_{2ft} = \frac{U_m}{\sqrt{2}}$$

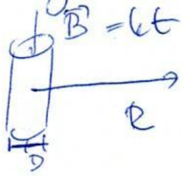
$$U^2_{2ft} = \frac{\pi^4R^4B^2f^2}{T^2}\underbrace{\int_0^T \sin^2(2\pi f t)\, dt}_{\frac{T}{2}} = \frac{\pi^4R^4B^2f^2}{2}$$

$$U_{2ft} = \frac{\pi^2R^2Bf}{\sqrt{2}}$$

amplituda proudu (vn. odpor Voltmetru je r)

$$I_{max} = \frac{\pi^2R^2Bf}{r}$$

Př. 3.3.12 železný ~~kruh~~ plechový válec



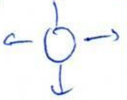
Najděte intenzitu elektrického el. pole ve vzd. R

$$\int_A^B \vec{E}\cdot d\vec{l} = U_{AB} \qquad \oint_A^A \vec{E}\cdot d\vec{l} = 0 \text{ v elstat}$$

v mag. ne

$$\oint \vec{E}\, d\vec{l} = -\frac{\partial\int \vec{B}\, d\vec{S}}{dt}$$

el pole je buď



nebo

nemůže, rozpor s Gauss. v.

$$\int \text{rot}\,\vec{E}\cdot d\vec{S} = -\frac{\partial\int B\, d\vec{S}}{dt}$$

$$\text{rot}\,\vec{E} + \frac{\partial\vec{B}}{\partial d} = \vec{0}$$

$$E 2\pi R = \frac{-\partial\left(kt\pi\frac{D^2}{4}\right)}{\partial t} = -k\pi\frac{D^2}{4}$$

$$E = -\frac{kD^2}{8R}$$

⑩

$$\Phi_1 = L_1 \overset{I}{\cancel{\Phi}}_1 + M_{12} \overset{I}{\cancel{\Phi}}_2$$

$$\Phi_2 = M_{21} \cancel{\Phi} I_1 + L_2 \cancel{\Phi} I_2$$

vzájemná indukčnost   vlastní indukčnost

$$Q_1 = \boxed{C_{11}} \varphi_1 + C_{12} \varphi_2$$

$$Q_2 = C_{21} \varphi_1 + \boxed{C_{22}} \varphi_2$$

infenenční koef.   kapacitní koef.

$$\varphi = A\,Q$$

potenciálové koef.

pro 1 smyčku: $\Phi = LI$

$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = -L\frac{\partial I}{\partial t}$$

$\oint \vec{E}\cdot d\vec{l} = 0$ platí i pro stacionární el. pole

Př. 3.4.7. dlouhá válcová cívka. Chceme vyjádřit pro indukčnost.

$$\Phi = LI \Rightarrow L = \frac{\Phi}{I} = \frac{\mu \cancel{I} N^2 \cdot S}{l\cdot \cancel{I}} = \boxed{\frac{\mu N^2 S}{l}}$$

$$\frac{\Phi}{N} = \oint \vec{B}\cdot d\vec{S} = BS = \frac{\mu I N}{l}\cdot S$$

pro solenoid platí: $\vec{B} = \frac{\mu I N}{l}$

Př. 3.4.1.

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int \frac{\vec{i}(\vec{r}')}{R}\,dV$$

pro vodič, kterým protéká proud

$$\vec{A}(R) = \frac{\mu I}{4\pi}\int \frac{d\vec{l}}{R}$$

$$\vec{R} = \vec{r} - \vec{r}'$$

$$L = \frac{\Phi}{I} = \frac{\int \vec{B}\cdot d\vec{S}}{I} = \frac{\int \operatorname{rot}\vec{A}\cdot d\vec{S}}{I} = \frac{\oint \vec{A}\cdot d\vec{l}}{I}$$

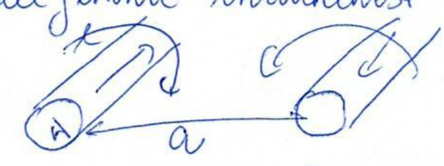
$$\vec{B} = \operatorname{rot}\vec{A}$$

$$= \frac{\mu_0 \cancel{I}}{4\pi \cancel{I}} \oint \frac{d\vec{l}\cdot d\vec{l}}{R}$$

Př. 3.4.3. dvě smyčky dokažte, že $M_{12} = M_{21}$

jedno $d\vec{l}$ bude u jedné cívky a druhé $d\vec{l}$ u druhé

Př. 3.4.11. vzájemná indukčnost dvou vodičů



1. ~~vlastní indukčnost~~ indukčnost

~~Kta~~ plochou mezi vodiči na jednotku délky

$$M = \frac{\Phi}{I} =$$

$$\Phi_L = \frac{\Phi}{l} = \int_r^{r+a} \frac{\vec{B}\cdot d\vec{S}}{l} = \frac{1}{l}\int_{0}^{l}\int_r^{r+a} \frac{\mu_0 I}{2\pi R}\cdot d\vec{l}\cdot d\vec{R}$$

$$= \int_r^{r+a} \frac{\mu_0 I}{2\pi R}\cdot d\vec{R} = \frac{\mu_0 I}{2\pi}\int_r^{r+a} \frac{1}{R}\,d\vec{l}$$

$$= \frac{\mu_0 I}{2\pi}\left[\log|R|\right]_r^{r+a}$$

$W = \frac{1}{2}LI^2 = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dV$

B nekonečně dlouhého vodiče:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B\cdot 2\pi R = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi R}$$

$= \frac{\mu_0 I}{2\pi}\cdot \log\frac{r+a}{r}$ – příspěvek 1 vodiče

jsou 2 a příspěvky se sčítají $\Rightarrow \Phi = 2\frac{\mu_0 I}{2\pi}\cdot\log\frac{r+a}{r}$

$$= \frac{\mu_0 I}{\pi}\log\frac{r+a}{r}$$

$$M = \frac{\mu_0 I}{\pi I}\log\frac{r+a}{r} = \frac{\mu_0}{\pi}\log\frac{r+a}{r}$$

$$W = \int_0^Q UI\,dt$$

pro kondenzátor: $W = \int_0^Q U\,dQ = \frac{1}{C}\int Q\,dQ = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QU = \frac{1}{2}CU^2$

$$\frac{dQ}{dt} = I \qquad U\,dt = -L\,dI$$

$$W_m = \int UI\,dt = L\int_0^I I\,dI = \frac{1}{2}LI^2 = \frac{1}{2}I\Phi = \frac{1}{2}\frac{\Phi^2}{L}$$

analogicky $W = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,dV \qquad W_m = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dV$

v předchozím příkladu $\qquad B = \mu H$

$$W_m = \frac{1}{2}\int\frac{B^2}{\mu}V = \frac{1}{2}\int_0^r \frac{\mu^2 I^2}{4\pi^2}\cdot\frac{R^2}{r^4}\cdot\frac{1}{\mu}\,l 2\pi R\,dR = \frac{1}{2}\cdot\frac{\mu I^2}{4\pi R}\cdot\frac{l 2\pi}{r^4}\int_0^r R^3\,dR$$

$$= \frac{\mu I^2 l}{16\pi}$$

$$dV = l 2\pi R\,dR$$

$$\oint \vec{B}\cdot d\vec{l} = \mu I\cdot\frac{\pi R^2}{\pi r^2}$$

$$B\,2\pi R = \mu I\frac{R^2}{r^2}$$

$$B = \frac{\mu I}{2\pi}\frac{R}{r^2}$$

$$W_m = \frac{\mu I^2 l}{16\pi} = \frac{1}{2}LI^2$$

$$L_{celk.} = \frac{2\mu I^2 l}{16\pi} = \frac{\mu l}{8\pi}$$

$$L_1 = \frac{L_{celk.}}{l} = \frac{\mu}{8\pi}$$

$$L = 2\cdot L_1 = \frac{\mu}{4\pi}$$

$$L = \frac{\mu}{4\pi}\left(1 + 4\log\left(\frac{r+a}{r}\right)\right)$$

Př. 4.1.11. Indukčnost na jednotku délky koaxiálního vedení

$$L = \frac{\mu_0}{4\pi}\left(\frac{1}{2} + 2\log\left(\frac{r_2}{r_1}\right)\right)$$

proud teče jen jedním $\longrightarrow \cdot\frac{1}{2}$

od r do r+a $\longrightarrow$ od r_1 do r_2

Př. 3.5.2 energie mag. pole v toroidu

$\vec{B} =$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

$$B 2\pi R\cdot\cancel{N} = \mu_0 I N$$

$$B = \frac{\mu_0 I N}{2\pi R}$$